指數函數及其斜率(slope)

例: $y_1 = 2^x$

$$slope = \frac{dy}{dx} = \frac{\Delta y}{\Delta x} = \frac{2^{x+\Delta x} - 2^x}{(x+\Delta x) - x} = \frac{2^x(2^{\Delta x} - 1)}{\Delta x} = 2^x \frac{(2^{\Delta x} - 1)}{\Delta x} = (0.693)\ 2^x$$

Note that: $\frac{(2^{\Delta x}-1)}{\Delta x} = slope\ at\ x = 0$

You may test this result with a calculator using a small Δx value like 0.001.

同理 $y_2 = 3^x$

$$slope = \frac{dy}{dx} = \frac{\Delta y}{\Delta x} = 3^x \frac{(3^{\Delta x} - 1)}{\Delta x} = (1.099)\ 3^x$$

同理 $y_3 = 2.7^x$

$$slope = \frac{dy}{dx} = \frac{\Delta y}{\Delta x} = 2.7^x \frac{(2.7^{\Delta x} - 1)}{\Delta x} = (0.993)(2.7)^x$$

可找到一個實數 e $(e \cong 2.71828)$ 使 $y_4 = e^x$

且 $slope = \frac{dy}{dx} = \frac{\Delta y}{\Delta x} = e^x \frac{(e^{\Delta x}-1)}{\Delta x} = (1)e^x$ 即 $\frac{dy}{dx} = y$

指數函數為單調函數，存在一反函數

使得 $\ln(y) = x$ 即 $\ln(e^x) = x$

類似 $\log(10^x) = x$

A useful expression: $e^x \cong 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots,\ -\infty < x < \infty$

Homework: Explain

$y_4 = e^{2x}$ $slope = \frac{dy}{dx} = \frac{\Delta y}{\Delta x} = (2)\ e^x$

微分方程式

求解 $\frac{dy}{dx} = -y$

即求滿足上式之 $y(x)$

猜 $y = Ce^{ax}$　　C, a 為待決定之常數

代入 $\frac{dy}{dx} = -y$

$\frac{d}{dx} Ce^{ax} = Cae^{ax} = -y = -Ce^{ax} \quad \therefore a = -1 \quad y = Ce^{-x}$

目前無法決定 C

同理

求解 $\frac{dy}{dx} = -ky,\ k = constant$

即求滿足上式之 $y(x)$

猜 $y = Ce^{ax}$　　a 為待決定之常數

代入 $\frac{dy}{dx} = -ky$

$\frac{d}{dx} Ce^{ax} = Cae^{ax} = -ky = -kCe^{ax} \quad \therefore a = -k \quad y = Ce^{-kx}$

How to make sure a curve is an exponential function?

$f(t) = e^{-kt} \qquad f(t + \Delta t) = e^{-k(t+\Delta t)} = e^{-kt}e^{-k\Delta t}$

$$\frac{f(t + \Delta t)}{f(t)} = e^{-k\Delta t} = constant\ if\ \Delta t = const.$$